Profit-sharing rules and taxation of multinational two-sided platforms

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Abstract

This paper analyzes taxation of a two-sided platform attracting users from different jurisdictions under two profit-sharing régimes: separate accounting and formula apportionment based on the number of users in the two countries. When corporate income tax rates are different in the two jurisdictions, the platform distorts prices and outputs in order to shift profit to the low tax country. When cross effects are present on both sides of the market, the platform has an incentive to increase output in the high tax country and decrease output in the low tax country under separate accounting. Under formula apportionment, the incentives are reversed, and the platform reduces output in the high tax country and increases output in the low tax country. We show that separate accounting always dominates formula apportionment for the platform, but that consumer surplus and tax revenues may be higher under formula apportionment than under separate accounting. In particular, consumers in the high tax country always favor separate accounting, whereas consumers of the low tax country prefer formula apportionment when the difference in corporate tax rates is not too high. Fiscal revenues of the high tax country are higher under Separate Accounting and fiscal revenues of the low tax country are higher under Formula Apportionment. Finally, we compute the equilibrium corporate tax rates under Separate Accounting and Formula Apportionment in a symmetric model of fiscal competition.

`JEL Classification Numbers:` H32, H25, L12, L14.

`Keywords:` Two-sided platforms, multinationals, corporate income taxation, separate accounting, formula apportionment.

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1 Introduction

Internet two-sided platforms often connect agents living under different fiscal jurisdictions. Facebook and Google users receive targeted advertising from companies headquartered outside their country of residence. Sellers and buyers on E Bay transact with agents living in different countries. Booking or Expedia users book flights and hotels all over the world. In these situations, the service of the two-sided platform (and its monetary value) cannot easily be ascribed to a specific fiscal jurisdiction, raising difficult issues when the jurisdictions involved charge different levels of corporate income taxes. In this paper, our objective is precisely to analyze different rules of taxation and value-sharing for two-sided platforms operating under different jurisdictions.

The problem of profit-sharing across multiple jurisdictions is not specific to multinational two-sided platforms. Any multinational firm, operating and creating value in different countries (or different states in a federation), will face a similar problem. In order to analyze the global value chain, one needs to clearly describe the activities and assets of the firm, analyze the exact sequence of operations leading to value creation, and ascribe each operation to a specific jurisdiction. In practice, multinational firms have the capacity to select transfer prices across divisions located in different jurisdictions in order to minimize their tax bill. In order to limit these incentives to evade corporate income taxation, the OECD has launched a global program on Base Erosion and Profit Shifting (BEPS) with two actions specifically designed to address the tax challenges in the digital economy (Action 1) and align transfer pricing with value creation (Action 8) (OECD, 2016).

Different methods for allocating the profit of the platform to different jurisdictions have been proposed. If the multinational declares profits separately in the different jurisdictions (Separate Accounting), profit is apportioned according to the profit declared in the different jurisdictions. Alternatively, profit can be apportioned using a formula based on other indicators of the activity of the firm (Formula Apportionment). Formula apportionment is used to allocate profit for corporate income taxation across members of federal states (states in the United States, provinces in Canada, cantons in Switzerland). In the United States, the formula uses three indicators: sales, assets and wages. Formula appointment has been advocated in the European Union for corporate income taxation of large corporations operating in different members of the union.[1] Separate Accounting (SA) and Formula Apportionment (FA) create different incentives for a multinational facing different corporate income tax rates. Under SA, the multinational has an incentive to use transfer prices to shift profit away from the high tax jurisdiction to the low tax jurisdiction. Under FA, profit shifting is irrelevant, but the multinational has an incentive to distort its activities in order to reduce the value of the indicators in high tax jurisdiction and increase them in low tax jurisdictions. In effect, FA imposes a specific tax on each of the activities of the multinational which are used to determine the apportionment formula.

In this paper, we analyze the behavior of a multinational two-sided platform under two different profit-sharing rules: Separate Accounting (SA) and, Formula Apportionment (FA). In the Separate Accounting régime, profit is allocated according to the declared profit in each

[1] Devereux and Lorenz (2008) estimate that this would multiply the amount of taxes collected on multinational firms inside the European Union by 2.

jurisdiction. In the Formula Apportionment régime, profit is allocated according to the number of participants in each jurisdiction.

In order to analyze the choices of the multinational two-sided platform in response to different corporate income tax rates, we need to make three simplifying assumptions. First, we consider a model with two jurisdictions, and assume that all users on the same side of the market belong to the same jurisdiction. Hence there are only two types of agents in the model: users in jurisdiction A on one side of the market and users in jurisdiction B on the other side of the market. Allowing for users in each jurisdiction to be on the two sides of the market would greatly complicate the analysis, resulting in four types of agents instead of two, without providing any additional insight. Second, we suppose that users on the two sides of the market are immobile and cannot move to other jurisdictions.

This assumption is clearly too strong, as platforms will try to locate their revenues in low tax countries if they can by forcing users to locate into these jurisdictions. This is particularly the case in Europe, with platforms like Google requiring advertisers to locate advertising contracts in Ireland. However, some countries, like the United Kingdom or Italy, are trying to prevent platforms to do so, by requiring advertising revenues to be declared in the country of origin of the advertiser. Introducing mobility of users would clearly increase the distortions in the Separate Accounting régime but not under Formula Apportionment. In order to keep the comparison clear, we have decided to leave aside the issue of mobility of users in the current analysis. Third, we only consider the operation of the platform given a technology, abstracting away from investment and intellectual property costs. We do not model the upstream investment and technology choices of the platform, and do not consider the incentives of the platform to shift profit by selecting transfer prices for the use of the algorithm or technology. The analysis of these incentives is standard and follows well-known arguments. Introducing them in the model would complicate the analysis without providing additional insight.

We first analyze the distortions in output, measured by the number of users, under the two régimes. Under SA, when externalities are symmetric, the multinational expands output in the high tax country and reduces output in the low tax country. The intuition for this unexpected result is as follows. In the absence of externalities between the two markets, no distortion in output would appear under SA. The positive consumption externality of the number of users on the other side of the market is the driving force behind the distortion. In order to shift profit to the low tax country, the platform can either choose to increase or decrease its output with respect to the optimal output. Increasing output in the high tax country has a positive externality on the other side of the market, thereby increasing profit in the low tax country. Similarly, decreasing output in the low tax country has a negative externality on the other side of the market, thereby decreasing profit in the high tax country. Overall, we find that the platform always has an incentive to increase output in the high tax country, and chooses to decrease output in the low tax country when the difference in corporate tax rates is not too high. When the difference in corporate tax rates becomes high, the increase in output in the high tax country becomes so large that it induces the platform to increase output on the other side of the market through the positive externality.

However, when externalities only affect one side of the market, the results change dramatically. Suppose that externalities only flow from country A to country B. When country B is the low tax country, decreasing output in country B cannot be used to shift profit. Hence, as the tax rate in country A increases, the platform will still choose to expand output in country A, but will not limit output in country B. Instead, as positive externalities, it will also expand output in country B. Hence an increase in the corporate tax rate of country A will induce the platform to increase its output in both jurisdictions.

By contrast, under FA, the multinational decreases output in the high tax country and expands output in the low tax country. This intuitive result stems from the fact that the multinational wants to reduce the profit apportioned to the high tax country and increase the profit apportioned to the low tax country. Notice however that, when the difference in corporate tax rates becomes large, the decrease in output in the high taxation jurisdiction may induce a decrease in the output of the low tax jurisdiction because of thee externalities. We thus obtain an interesting difference in the output choices of the platform under SA and FA. Under SA, output is higher in the high taxation region than in the low taxation region, whereas the opposite is true under FA. An increase in the corporate tax rate leads to an increase in output under SA but a reduction in output under FA. This direct effect may however be overcome by an indirect effect due to externalities when the difference in corporate tax rates is high, leading to an increase in output when the corporate tax rates increases for the low taxation region under FA, and a decrease in output when the corporate tax rate increases under SA.

We then analyze how the platform's profit and the fiscal revenues of the two countries differ in the two régimes. We find that the platform is always better off under SA than under FA for any level of corporate taxation. The distortions generated by Formula Apportionment always exceed the distortions linked to externalities under Separate Accounting. Fiscal revenues of the high tax country are higher under Separate Accounting and fiscal revenues of the low tax country are higher under Formula Apportionment. Finally, the computation of fiscal revenues under the optimal choice of the platform allows us to study a model of fiscal competition between two symmetric countries. We show that under FA, incentives to shift profit are not strong enough to prevent both countries to set the highest level of corporate income taxes. Under SA however, when externalities are large enough, the platform is able to shift profit, reducing the equilibrium level of corporate income taxes to an interior value.

The rest of the paper is organized as follows. We discuss the related literature in the next subsection. We present the model in Section 2. In Section 3, we analyze output and price distortions of the platform under the two régimes. Section 4 si devoted tot he study of profits and tax revenues and contains our analysis of fiscal competition among two symmetric countries. We discuss extensions of the model beyond the linear case in Section 5. Section 6 concludes.

1.1 Relation to the literature

This paper is related to two different strands of the literature: the literature on taxation of two-sided platforms and the literature on formula apportionment. Optimal taxation of two-sided monopolistic platforms has been studied by Kind et al. (2008, 2009, 2010, 2013) and Bourreau, Caillaud and de Nijs (2018). The main focus of these papers is not on corporate income taxes but on unit and ad valorem taxes. The studies of Kind et al. (2008, 2009, 2010, 2013) have generated two main results. First,they show that ad valorem taxes (like VAT) do not necessarily dominate unit taxes. The classical result in public finance on the domination of ad valorem taxes no longer holds for two-sided markets. Second, the price of a good may decrease with the ad valorem tax. The introduction of a tax on the value added for one side of the market can lead to a change in the entire business model of the platform. For example, the increase in VAT on the price of access for users could induce the platform to set a zero price for Internet access and switch all its revenues to the advertisers side. Bourreau, Caillaud and de Nijs (2018) supplement the model of the two-sided platform by considering data collection and letting consumers select the flow of data uploaded to the platform. They compare taxes levied on the flow of data uploaded by users with taxes paid by advertisers, and analyze the interaction between VAT and taxes based on the flow of data.

Schindler and Schjelderup (2010) study multinational two-sided platforms but in a very different context, with one parent firm selling goods to affiliates in different countries, assuming positive externalities between the good sold by the multinational firm and the goods sold locally by all the affiliates. They study how transfer pricing is affected by the externalities and analyze the distortions due to differences in corporate tax rates.

In a paper more closely related to our analysis, Kotsogiannis and Serfes (2010) address the issue of taxation with multi-sided market from the point of view of tax competition between countries. They consider competition between two countries that choose two tax instruments and the provision of local public goods, taking into account that each instrument is designed to attract both sides of a two-sided market, namely consumers and businesses. Consumers are located along a Hotelling segment, and two platforms are formed at both ends of the segment. Each firm chooses in which platform to go, and consumers choose to go on either platform based on the number of companies on each platform and the distance to the consumer platform. The time sequence of the model is as follows: the two jurisdictions first choose their levels of public good, and their level of taxation, and consumers and businesses simultaneously choose their platforms. Suppose that jurisdiction A provides more public goods that platform B. If the difference is large, vertical differentiation between platforms is important, and each platform specializes in a segment of the population. If the difference is small, competition between platforms is intense, and it is possible that all consumers and all businesses meet on a single platform. Comparative statics results show that an increase in externalities between the two sides of the market may lead to a decrease in the tax rate in both jurisdictions, an increase in the number of firms on platform A and a decrease in the number of firms on platform B. The model of Kotsogiannis and Serfes (2010) differs from ours in several respect. First they consider perfectly mobile users on the two sides of the market, second they assume that two platforms compete, one in each of the jurisdictions. Finally, they consider taxation on firms and consumers whereas we analyze corporate income taxes paid by the monopolistic platform.

The literature on formula apportionment started with a paper by Gordon and Wilson (1986). They show that the formula used in the United States, which puts positive weight on sales, wages and assets induces distortions in the optimal choice of inputs by the firms. In addition, it results in discriminatory treatment of companies in the same jurisdiction as they will face different effective tax rates. They advocate using an accounting system which replicates the separate accounting in each state. Anand and Sansing (2000) provide a clear account of the history of formula apportionment in the United States They analyze a model where two states bargain over the weights to place on different indicators and show that the weights placed on sales and inputs are typically inefficient in a decentralized equilibrium. Nielsen, Raimondos Moller and Schjelderup (2003) compare SA and FA in a model where transfer prices are used as a way to manipulate the behavior of a subsidiary in an oligopolistic market. Because transfer prices have an additional role in SA, where they allow for profit shifting, the incentives to manipulate are higher under SA than under FA. Kind, Midelfart and Schjelederup (2005) extend the model by considering a first stage of tax competition where two countries simultaneously select their corporate income tax rate to maximize fiscal revenues. The main result of the analysis shows that when transportation costs are low (countries are more integrated), equilibrium tax rates are higher under FA than under SA whereas the opposite holds when transportation costs are high. Nielsen, Raimondos-Moller et Schjelderup (2010) analyze capital investment decisions of a multinational under the two régimes of SA and FA. They analyze the effect of an increase in the tax rate on the capital accumulation of the multinational around symmetric tax rates, and show that the effect is higher for FA than for SA. The rationale is that capital levels not only affect the profit of the multinational but also the apportionment formula. This result echoes our result on output choices by the multinational two-sided platform, where FA leads to larger manipulations than SA. Finally, Gresik (2010) compares SA and FA when the production cost of the intermediate output is privately known by the multinational. Under FA, as transfer prices are irrelevant, the equilibrium tax rate only depends on the average production cost. Under SA, equilibrium tax rates are more complex as the multinational manipulates transfer prices. Gresik (2010) shows that equilibrium tax rates are higher under SA and that all firms prefer FA.

2 The model

2.1 Utilities of users, and pre-tax profit of the platform

We consider a monopolistic two-sided platform, with two distinct group of users. Users is a generic term, which represent different types of agents according to the specific platform. Users can represent advertisers and consumers (in the context of search engines or digital social media), buyers and sellers (in the context of auctions or travel reservation platforms), or can be drawn from the same population (in the context of peer-to-peer transactions).

The two groups of users are located in different jurisdictions, denoted A and B. Users pay a fixed fee to participate in the platform, p_A or p_B according to their location. They are immobile and cannot move to the other jurisdiction. We consider only the extensive margin of participating in the platform. The volume of use of the platform is supposed to be fixed and identical across users. In each jurisdiction, the utility of users is the sum of two components: an idiosyncratic utility for the platform, which is heterogeneous across users, and an externality term, which is increasing in the number of users on the other side of the market. Formally, letting x_A and x_B denote the number of users participating in the platform in jurisdictions A and B, the utility of users in the two jurisdictions is given by

$$\begin{aligned} U_A &= \theta + u_A(x_B) - p_A, \\ U_B &= \eta + u_B(x_A) - p_B \end{aligned}$$

where θ is distributed according to a continuous distribution with full support F_A on $[\underline{\theta}, \overline{\theta}]$, u_A is weakly increasing in x_B, η is distributed according to a continuous distribution with full support F_B on $[\underline{\eta}, \overline{\eta}]$, u_B is weakly increasing in x_A. We let y_A and y_B denote the consumers who are indifferent between accessing the platform or not. The consumer surpluses in the two jurisdictions are then given by

$$\begin{aligned} C_A &= \int_{y_A}^{\overline{\theta}} [\theta + u_A(x_B) - p_A] dF_A, \\ C_B &= \int_{y_B}^{\overline{\eta}} [\eta + u_B(x_A) - p_B] dF_B \end{aligned}$$

For most of the analysis, we will restrict attention to a linear model where the idiosyncratic shocks are drawn from a uniform distribution on $[0, 1]$ and the externalities are given by $u_A(x_B) = \beta x_B$ and $u_B(x_A) = \alpha x_A$. In this linear formulation, the indifferent consumers are given by

$$\begin{aligned} y_A &= p_A - \beta x_B, \\ y_B &= p_B - \alpha x_A \end{aligned}$$

and demand by

$$\begin{aligned} x_A &= 1 - y_A &= 1 - p_A + \beta x_B, \\ x_B &= 1 - y_B &= 1 - p_B + \beta x_A. \end{aligned}$$

Consumer surpluses are given by

$$\begin{aligned} C_A &= \frac{x_A^2}{2}, \\ C_B &= \frac{x_B^2}{2} \end{aligned}$$

and the inverse demand functions are

$$\begin{aligned} P_A(x_A, x_B) &= 1 - x_A + \beta x_B, \\ P_B(x_A, x_B) &= 1 + \alpha x_A - x_B. \end{aligned}$$

The platform provides a service valued by each user, and charges discriminatory prices in the two jurisdictions p_A and p_B. There is a one-to-one connection between the prices chosen by the monopolistic platform and the number of participants x_A and x_B and it will prove easier to write the profit in terms of numbers of users. We suppose that operating costs of the platform are negligible so that the pre-tax profit in each jurisdiction is given by

$$\begin{aligned} V_A &= x_A P_A(x_A, x_B), \\ V_B &= x_B P_B(x_A, x_B) \end{aligned}$$

and the total pre-tax profit as

$$V = V_A + V_B = x_A(1 - x_A + \beta x_B) + x_B(1 - x_B + \alpha x_A).$$

The first order conditions give

$$\begin{aligned} 2x_A - (\alpha + \beta)x_B &= 1, \\ -(\alpha + \beta)x_A + 2x_B &= 1. \end{aligned}$$

Concavity of the profit will obtain if $(\alpha + \beta) < 2$, resulting in the identical equilibrium quantities

$$x_A^* = x_B^* = \frac{1}{2 - (\alpha + \beta)}.$$

These equilibrium quantities will satisfy the market coverage constraint $x^* \leq 1$ if and only if

$$1 < (\alpha + \beta),$$

an assumption that we maintain throughout the analysis. Equilibrium prices are then given by

$$p_A^* = p_B^* = \frac{1}{2 - (\alpha + \beta)},$$

equilibrium profits of the platform by

$$V^* = \frac{2}{2 - (\alpha + \beta)^2},$$

and consumer surpluses

$$C_A^* = C_B^* = \frac{1}{2(2 - (\alpha + \beta)^2)}.$$

2.2 Profit-sharing rules and post-tax profit of a platform

We suppose that the two jurisdictions charge corporate income tax rates t_A and t_B. We consider three different rules for apportioning the profit of the platform to the two jurisdictions.

2.2.1 Separate accounting

In *Separate Accounting (SA)*, the platform declares separately the profit made in each jurisdiction, and pays taxes in each jurisdiction based on this profit. This amounts to apportioning profit based on the profit made in each jurisdiction. The post-tax profit of the platform is then given by

$$\Pi = (1 - t_A)V_A + (1 - t_B)V_B.$$

The fiscal revenues of the two countries under separate accounting are computed as

$$\begin{aligned} T_A &= t_A V_A, \\ T_B &= t_B V_B. \end{aligned}$$

2.2.2 Formula Apportionment

In the régime of *Formula Apportionment (FA)*, the profit is apportioned according to the number of users in the two jurisdictions. The post-tax profit of the platform is given by

$$\Pi = V(1 - t_A \frac{x_A}{x_A + x_B} - t_B \frac{x_B}{x_A + x_B}).$$

We compute the fiscal revenues of the two countries as

$$\begin{aligned} T_A &= t_A \frac{x_A}{x_A + x_B} V, \\ T_B &= t_B \frac{x_B}{x_A + x_B} V \end{aligned}$$

3 Output and price distorsions

In this section, we study how the optimal choices of the platform – the prices charged to users and the number of users in each jurisdiction – are affected by the corporate tax rates in each jurisdiction under the two régimes of profit sharing.

3.1 Separate accounting

In the régime of separate accounting, it will prove useful to define $r_A = 1 - t_A, r_B = 1 - t_B$. The post-tax profit of the platform is given by

$$\Pi = r_A(1 - x_A + \beta x_B) + r_B(1 - x_B + \beta x_A),$$

resulting in the first order conditions

$$\begin{aligned} 2r_A x_A - (\beta r_A + \alpha r_B)x_B &= r_A, \\ -(\beta r_A + \alpha r_B)x_A + 2r_B x_B &= r_B \end{aligned}$$

Concavity requires that

$$4r_A r_B > (\beta r_A + \alpha r_B)^2.$$

and the equilibrium quantities are given by

$$\begin{aligned} x_A^* &= \frac{2r_A r_B + r_B(\beta r_A + \alpha r_B)}{4r_A r_B - (\beta r_A + \alpha r_B)^2}, \\ x_B^* &= \frac{2r_A r_B + r_A(\beta r_A + \alpha r_B)}{4r_A r_B - (\beta r_A + \alpha r_B)^2} \end{aligned}$$

Inspection of this formula shows that the equilibrium quantities do not depend on the absolute values of the tax rates t_A and t_B but only on the *relative tax rate*. We thus define $\lambda = \frac{r_A}{r_B}$ and compute

$$\begin{aligned} x_A^* &= \frac{2\lambda + (\beta\lambda + \alpha)}{4\lambda - (\beta\lambda + \alpha)^2}, \\ x_B^* &= \frac{2\lambda + \lambda(\beta\lambda + \alpha)}{4\lambda - (\beta\lambda + \alpha)^2} \end{aligned}$$

Observe that $x_A^* > x_B^*$ if and only if $\lambda < 1$. Furthermore, it appears that distortions from the optimum are due to the presence of externalities. If $\alpha = \beta = 0$, then the platform's ouptut choices in the two markets are independent, and are not affected by differences in corporate tax rates. Define

$$\begin{aligned} \underline{\lambda} &= \frac{2 - \beta - 2\alpha\beta - \sqrt{4 - 4\beta - 8\alpha\beta + \beta^2}}{2\beta^2}, \\ \overline{\lambda} &= \frac{2 - \alpha - 2\alpha\beta + \sqrt{4 - 4\alpha + \alpha^2 - 8\alpha\beta}}{2(\beta + \beta^2)} \end{aligned}$$

It is easy to check that, given that $1 > \alpha + \beta$, $\underline{\lambda} < 1 < \overline{\lambda}$. We now obtain the following Proposition

Proposition 1 *The equilibrium output choice of the monopolistic platform is given as follows.*

- *If* $\underline{\lambda} \geq \lambda$*, then*

$$\begin{aligned} x_A^* &= 1, \\ x_B^* &= \frac{1 + \alpha + \lambda\beta}{2} \end{aligned}$$

- *If* $\overline{\lambda} \geq \lambda \geq \underline{\lambda}$*, then*

$$x_A^* = \frac{2\lambda + (\beta\lambda + \alpha)}{4\lambda - (\beta\lambda + \alpha)^2},$$
$$x_B^* = \frac{2\lambda + \lambda(\beta\lambda + \alpha)}{4\lambda - (\beta\lambda + \alpha)^2}$$

- *If* $\lambda \geq \overline{\lambda}$*, then*

$$x_A^* = \frac{\lambda(1+\beta) + \alpha}{2\lambda},$$
$$x_B^* = 1.$$

Proposition 1 identifies three regions of parameters with different equilibria. When the ratio λ is small (the corporate tax rate is much higher in jurisdiction A than jurisdiction B), the platform chooses to cover all users in jurisdiction A. When the ratio λ is intermediate (the corporate tax rates are similar in the two regions), the platform chooses interior volumes and does not cover all users in any jurisdiction. When the ratio λ is high (the corporate tax rate is much higher in jurisdiction B than in jurisdiction A), the platform chooses to cover all users in jurisdiction B.

We thus observe that the platform chooses to cover the market in the jurisdiction with the highest tax rate. The reasoning underlying this behavior is easy to grasp. When there are positive externalities on the two sides of the market, the platform uses output to shift profits from the high taxation jurisdiction to the low taxation jurisdiction. By choosing high output in the high taxation jurisdiction, the platform reduces its profit in the high taxation jurisdiction. Because of the positive externalities, this also increases demand and hence the profits in the low taxation jurisdiction.

This intuition explains why, for extreme values of the corporate taxation rate, the platform may choose to serve a large volume of users and cover the market in the high taxation jurisdiction. However, for intermediate values of the corporate taxation rate, this effect may not be strong enough, so that an increase in the corporate tax rate in one jurisdiction may have ambiguous effects on the outputs of the two jurisdictions. To analyze this, we now consider explicitly the comparative statics effects of changes in λ on x_A^* and x_B^*.

Consider first the extreme regions, where $\lambda \leq \underline{\lambda}$ or $\lambda \geq \overline{\lambda}$. The volume in the high taxation jurisdiction is fixed, equal to 1 as the market is covered. The platform chooses its output in the low taxation jurisdiction, understanding that an increase in output raises the profit in the high taxation jurisdiction (through the externality) and in the low taxation jurisdiction. As the difference in taxation rates is reduced, the output in the low taxation jurisdiction goes up, closing the gap in volume between the two jurisdictions.

In the intermediate region where $\underline{\lambda} \leq \lambda \leq \overline{\lambda}$, a reduction in the difference in taxation rates has ambiguous effects on the output in the low and high taxation jurisdictions. This effect depends on the intensity of externalities across the two regions.

Consider first the case of *symmetric externalities* where $\alpha = \beta$. The equilibrium output levels are given by

$$x_A^* = \frac{(2+\alpha)\lambda + \alpha}{4\lambda - (\alpha(\lambda+1))^2},$$
$$x_B^* = \frac{\alpha\lambda^2 + (2+\alpha)\lambda}{4\lambda - (\alpha(\lambda+1))^2}.$$

Figure 1 illustrates the output levels for $\alpha = \beta = 0.3$ The blue curve depicts x_A^* and the orange curve x_B^*. As is apparent from the figure, x_A^* is decreasing in λ when $\underline{\lambda} \leq \lambda \leq 1$ and x_B^* is increasing in λ when $1 \leq \lambda \leq \overline{\lambda}$. In other words, *the output of the high taxation jurisdiction goes up when the difference between the corporate tax levels increases.* When externalities are symmetric, the platform has an incentive to increase the output in the high taxation jurisdiction, as this shifts profit towards the low taxation jurisdiction. Notice however that the output in the low taxation jurisdiction may either increase or decrease when the difference between the corporate tax levels increases. To understand this, note that in the low taxation jurisdiction, two effects are at play. On the one hand, the increase in output in the other jurisdiction leads to a positive shift in demand, pushing the output upwards. On the other hand, compared to the optimal (pre-tax) output choice, the platform has an incentive to decrease its output to lower profit in the high tax jurisdiction. The balance between these two effects leads the platform to reduce the output in the low taxation jurisdiction when the difference in corporate tax rates is small (and hence the demand shift effect is small compared to the profit shifting incentive.) It may however lead the platform to increase the output in the low level jurisdiction when the difference in corporate tax rates is high, and the demand shift effect dominates the profit shifting incentive.

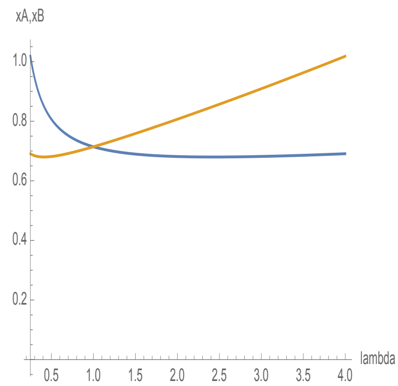

Figure 1: Output distortions under separate accounting and symmetric externalities

The comparative statics effect of a change in the corporate taxation rate is particularly stark at the point where $\lambda = 1$. Consider a small deviation in the corporate tax rate from the symmetric situation where both jurisdictions impose the same rate. We compute

$$\frac{\partial x_A^*}{\partial \lambda} = -\frac{4\alpha(1-\alpha^2)}{(4-\alpha^2)^2} < 0,$$
$$\frac{\partial x_B^*}{\partial \lambda} = \frac{4\alpha(1-\alpha^2)}{(4-\alpha^2)^2} > 0.$$

Hence, at the point where the two corporate tax rates are equal, an increase in the corporate tax rate in one country increases output in that country and reduces output in the other country.

Next consider a polar opposite case, where *externalities are one-sided.* Suppose that users in jurisdiction A create positive externalities on users in jurisdiction B ($\alpha > 0$) but that users in jurisdiction B do not create any externality on users of jurisdiction A ($\beta = 0$). A typical situation where externalities are one-sided is advertising on search engines and social networks. Users have little utility for advertisers whereas advertisers have a utility which is increasing in the number of users on the platform. In that case, we write optimal outputs as

$$x_A^* = \frac{2\lambda + \alpha}{4\lambda - \alpha^2},$$
$$x_B^* = \frac{2\lambda + \alpha\lambda}{4\lambda - \alpha^2}$$

Figure 2 illustrates the output levels for $\alpha = 0.3$. Again, the blue curve depicts x_A^* and the orange curve depicts x_B^*. It is easy to check that *both outputs are decreasing in* λ. In particular, we see that at $\lambda = 1$, an increase in the taxation level of jurisdiction A induces an increase in the output in both jurisdictions. To understand this fact, and the difference with the case of symmetric externalities, recall that the output choice x_B^* does not affect the demand in jurisdiction A. Hence the platform has no incentive to reduce output in region B to reduce profit in the high taxation jurisdiction. The profit shifting effect is absent when country B is the low taxation country, so that output is always increasing with the corporate tax rate in country A.

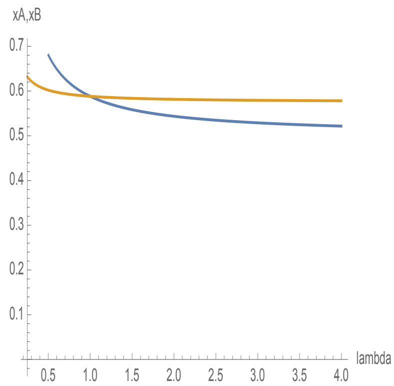

Figure 2: Output distortions under separate accounting and one-sided externalities

These output choices result in the following equilibrium prices set by the monopolistic platform

Proposition 2 *The equilibrium prices are set by the monopolistic platform as follows.*

- *If* $\underline{\lambda} \geq \lambda$, *then*

$$
\begin{aligned}
p_A^* &= \frac{\beta(1+\alpha+\lambda\beta)}{2}, \\
p_B^* &= \frac{1+\alpha-\lambda\beta}{2}
\end{aligned}
$$

- *If* $\overline{\lambda} \geq \lambda \geq \underline{\lambda}$, *then*

$$
\begin{aligned}
p_A^* &= \frac{2\lambda(\beta+1)-(\alpha+1)(\beta\lambda+\alpha)}{4\lambda-(\beta\lambda+\alpha)^2}, \\
p_B^* &= \frac{2\lambda(1+\alpha)-(\beta\lambda+\alpha)(\beta+1)\lambda}{4\lambda-(\beta\lambda+\alpha)^2}
\end{aligned}
$$

- *If* $\lambda \geq \overline{\lambda}$, *then*

$$
\begin{aligned}
p_A^* &= \frac{\lambda(1+\beta)-\alpha}{2\lambda}, \\
p_B^* &= \frac{\alpha[(1+\beta)\lambda+\alpha]}{2\lambda}
\end{aligned}
$$

Observe that when the market in jurisdiction A is covered, the price p_A^* is increasing in λ whereas the price p_B^* is decreasing in λ. The platform has an incentive to set a very low price in the high tax jurisdiction whereas it will charge a high price to users in the low tax jurisdiction. Hence differences in tax rates give another rationale for discriminatory pricing on the two sides of the market. Symmetrically, when the market in jurisdiction B is covered, the price p_A^* is increasing in λ and the price p_B^* is decreasing in λ.

The same result obtains for the intermediate range of corporate tax levels as illustrated by the following two Figures. Figure 3 depicts the prices p_A^* (in blue) and p_B^* (in orange) in the symmetric externalities case when $\alpha = \beta = 0.3$. Figure 4 graphs the prices p_A^* and p_B^* in the one-sided externality market when $\alpha = 0.3, \beta = 0$. Notice that, as opposed to the case of output levels, the effect of a change in λ on equilibrium prices p_A^* and p_B^* is the same in the symmetric externalities and one-sided externalities situations. In both cases, the price of jurisdiction A is decreasing with the corporate tax rate of region A whereas the price in jurisdiction B is increasing in the corporate tax rate of region A.

Finally, remember that the consumer surplus in every region is proportional to the square of the output. Hence, the comparative statics effects of changes in the tax rate on consumer surplus are equal to the comparative statics effects of changes in the tax rate on output. In the symmetric externalities case, an increase in the difference in corporate tax rates leads to an

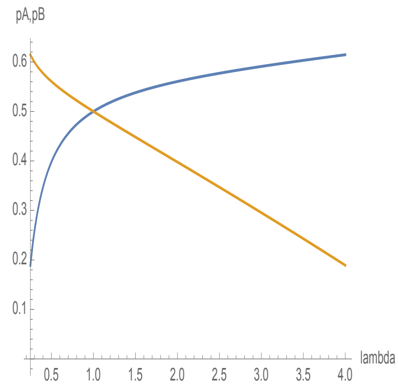

Figure 3: Price distortions under separate accounting and symmetric externalities

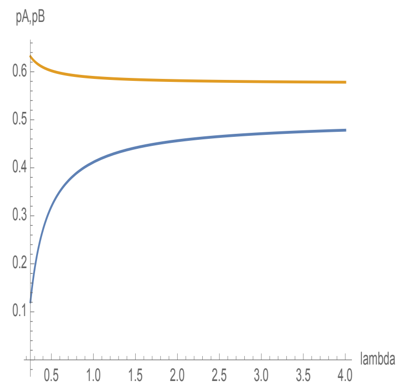

Figure 4: Price distortions under separate accounting and one-sided externalities

increase in consumer surplus in the high taxation region, a reduction in consumer surplus in the low taxation region when the difference between corporate tax rates is small, and may induce an increase in the consumer surplus in the low taxation region when the difference between corporate tax rates is high. In the case of one-sided externalities, an increase in the corporate tax rate of the region producing externalities has a positive effect on consumer surplus in both jurisdictions. An increase in the corporate tax rate of the region producing no externalities reduces consumer surplus in both regions.

3.2 Formula apportionment

To compute the optimal choice of the platform under formula apportionment, let us denote X the total output of the platform and μ the share of users in jurisdiction A.

$$X = x_A + x_B,$$
$$\mu = \frac{x_A}{x_A + x_B}$$

With this change of variables, the post-tax profit of the platform is given by

$$\begin{aligned}\Pi &= [\mu X(1-\mu X+(1-\mu)\beta X)+(1-\mu)X(1-(1-\mu)X+\mu\alpha X)](1-\mu t_A-(1-\mu)t_B),\\ &= X[1-\mu^2 X-(1-\mu)^2 X+(\alpha+\beta)\mu(1-\mu)X](1-\mu t_A-(1-\mu)t_B)\\ &= X[1-X(1-\mu(1-\mu)(2+\alpha+\beta))](1-\mu t_A-(1-\mu)t_B)\end{aligned}$$

Notice that, from the point of view of the platform, the two jurisdictions are symmetric, and the only relevant externality parameter is the sum of the externalities on the two sides, $\gamma=\alpha+\beta$. We can thus treat the two jurisdictions symmetrically,. Suppose that jurisdiction A is the high tax jurisdiction, $t_A \geq t_B$. We then consider the problem

$$\max_{X,\mu} X[1-X(1-\mu(1-\mu)(2+\gamma))](1-\mu t_A-(1-\mu)t_B)$$

subject to

$$\begin{aligned}0 &\leq X &&\leq \min\{\frac{1}{\mu},\frac{1}{1-\mu}\},\\ 0 &\leq \mu \leq 1.\end{aligned}$$

In order to get some insights into the solution to the maximization problem, we consider the relaxed problem, ignoring the constraint $X \leq \min\{\frac{1}{\mu},\frac{1}{1-\mu}\}$, and compute

$$\frac{\partial \Pi}{\partial X}=(1-\mu t_A-(1-\mu)t_B)[1-2X(1-\mu(1-\mu)(2+\gamma))].$$

Let

$$f(\mu)\equiv\frac{1}{2(1-\mu(1-\mu)(2+\gamma))}.$$

Notice that

$$\frac{\partial \Pi}{\partial X}>0 \Leftrightarrow X<f(\mu).$$

Similarly, compute

$$\frac{\partial \Pi}{\partial \mu}=X[-(t_A-t_B)+X[(t_A-t_B)+\mu^2(2+\gamma)(t_A-t_B)+(1-2\mu)(2+\gamma)(1-t_B)]].$$

Let

$$g(\mu)\equiv\frac{t_A-t_B}{t_A-t_B+\mu^2(2+\gamma)(t_A-t_B)+(1-2\mu)(2+\gamma)(1-t_B)}.$$

Notice that

$$\frac{\partial \Pi}{\partial \mu} > 0 \Leftrightarrow X > g(\mu).$$

Figure 5 is a phase diagram representing the two equations $f(\mu)$ and $g(\mu)$ for the parameter values $t_A = 0.7, t_B = 0.2, \gamma = 0.6$. It shows that there exists a unique saddle-point, corresponding to the maximum of the profit function Π. This unique saddle-point is characterized by the solution to the system of equations

$$\begin{aligned} \mu^2(t_A - t_B) - 2\mu(1 - t_B) + (1 - t_A) &= 0, \\ X[1 - \mu(1 - \mu)(2 + \gamma)] &= \frac{1}{2}. \end{aligned}$$

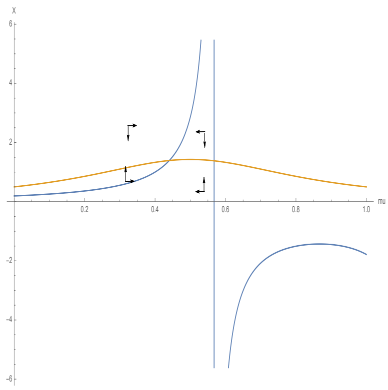

Figure 5: Saddle-point for profit under formula apportionment

Notice that the optimal fraction μ^* is *always smaller than one half* when jurisdiction A is the high taxation jurisdiction, and is independent of γ. It only depends on the tax rates t_A and t_B in the two jurisdictions. In fact, as in the case of separate accounting, the optimal output choices of the platform only depend on the *relative tax rate* λ and are given by

$$\begin{aligned} \mu^* &= \frac{1 - \sqrt{1 - \lambda(1 - \lambda)}}{1 - \lambda}, \\ X^* &= \frac{(1 - \lambda)^2}{(2[(1 - \lambda)^2 - (2 + \gamma)[(1 + \lambda)(\sqrt{1 - \lambda + \lambda^2} - 1 - \lambda^2)]]} \end{aligned}$$

yielding finally

$$x_A^* = \frac{(1 - \sqrt{1 - \lambda(1 - \lambda)})(1 - \lambda)}{2[(1 - \lambda)^2 - (2 + \gamma)[(1 + \lambda)(\sqrt{1 - \lambda + \lambda^2} - 1 - \lambda^2)]]}, \tag{1}$$

$$x_B^* = \frac{(\sqrt{1 - \lambda - (1 - \lambda)} - \lambda)(1 - \lambda)}{2[(1 - \lambda)^2 - (2 + \gamma)[(1 + \lambda)(\sqrt{1 - \lambda + \lambda^2} - 1 - \lambda^2)]]}, \tag{2}$$

We now check whether the output x_A^*, x_B^* satisfy the conditions: $x_A^* \leq 1$ and $x_B^* \leq 1$. It is easy to see that x_A^* and x_B^* are both increasing in γ. Furthermore, straightforward computations show that, at $\gamma = 0$, $x_A^* < 1$ and $x_B^* < 1$ for all positive λ. Hence there exists an upper bound on externalities, $\overline{\gamma}$ such that $x_A^* \leq 1$ and $x_B^* \leq 1$ for any $\gamma < \overline{\gamma}$ and any positive λ. We now suppose that externalities are bounded above by $\overline{\gamma}$ so that the solution to the relaxed maximization problem is also the solution to the maximization problem.

Proposition 3 *There exists a value of externalities $\overline{\gamma}$ such that for all $\gamma \leq \overline{\gamma}$, the optimal outputs under Formula Apportionments are given by equations (1) and (2).*

We now analyze how the optimal outputs depend on the difference in corporate tax rates. It is easy to see that μ is *decreasing* in λ. The share of users of the platform in jurisdiction A is *decreasing* in the difference in corporate tax rates between region A and region B. Total output X is non-monotonic in the difference in corporate tax rates. As illustrated in Figure 6, for $\gamma = 0.6$, total output is maximized at $\lambda = 1$, when the corporate tax rates are equal in the two regions. Any difference in corporate tax rates induces the platform to distort outputs, reducing output in the high taxation region and increasing output in the low taxation region, but the reduction of output in the high taxation region dominates the increase in the low taxation region.

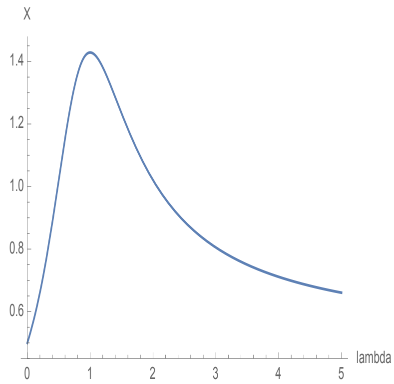

Figure 6: Total output under formula apportionment

Next consider the output x_A^* in jurisdiction A. As the corporate tax rate in jurisdiction A goes down (λ increases), *output increases as long as region A remains the high taxation region.* At some point, when the tax rate in jurisdiction B becomes sufficiently high, the platform stops increasing the output in region A, and instead chooses to reduce output in both regions, albeit at a higher rate in region B than in region A. This is illustrated in Figure 7 for the case where $\gamma = 0.6$. Observe that the maximal output in region A is achieved when region A is the low taxation region, but when the difference in corporate tax rates is small. In fact, when $\gamma = 0.6$, output in region A is maximized when $\lambda \sim 1.5$. When $\lambda = 1$, output is increasing in λ.

In a symmetric way, Figure 8 shows how the output x_B^* of jurisdiction B changes with λ. Output is first increasing in λ and the maximum output is obtained for $\lambda \sim \frac{1}{1.5}$. Output is then

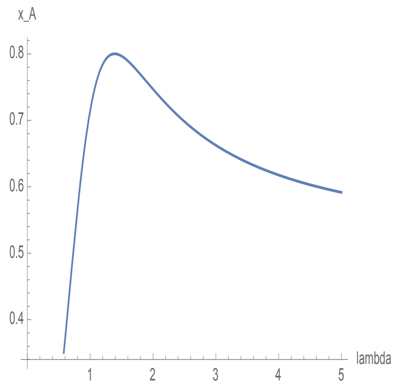

Figure 7: Output in region A under formula apportionment

decreasing in λ. In particular, when $\lambda = 1$, the output of jurisdiction B is decreasing in λ.

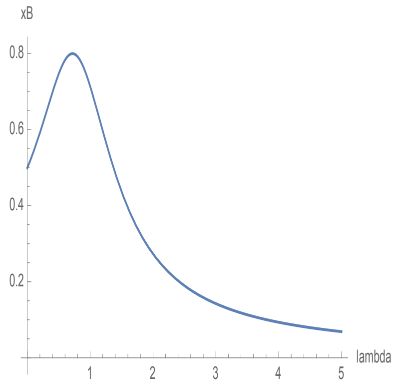

Figure 8: Output in region B under formula apportionment

The comparative statics effects of changes in the corporate tax rate on prices reflects the effect of changes in corporate tax rate on output. When $\lambda < 1$ (region A is the high tax jurisdiction), a reduction in the corporate tax rate t_A increases x_A and either reduces x_B (when λ is close to 1) or increases x_B but at a rate smaller than the rate of increase in x_A. Hence in total, a reduction in the corporate tax rate in jurisdiction A results in a *decrease* in the price p_A. When λ is high, and A is the low taxation region, a reduction in the corporate tax rate t_A will eventually lead to a reduction in both outputs x_A^* and x_B^*. This will eventually result in an increase in the price p_A when the corporate tax rate in region A is sufficiently small. This is illustrated in Figure 9 for $\alpha = \beta = 0.3$.

Finally, as consumer surplus is an increasing function of quantities, an increase in the corporate tax rate in region A reduces consumer surplus in that region when λ is small, but will eventually increase the consumer surplus when λ becomes high. For intermediate values of λ, quite naturally, an increase in the corporate tax rate of region A results in a reduction in

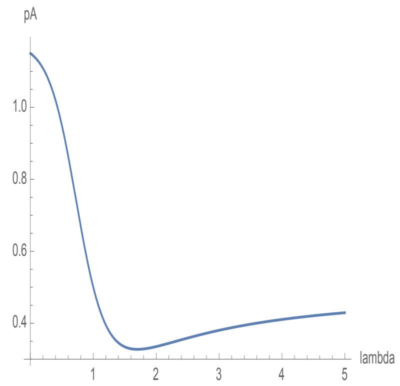

Figure 9: Price in region A under formula apportionment

consumer surplus in that region and an increase in consumer surplus in the other jurisdiction.

3.3 Comparison between the two régimes

We summarize our findings by comparing optimal outputs and prices in the two régimes of Separate Accounting and Formula Apportionment. Figure 10 illustrates the equilibrium output of region A as a function of λ when $\alpha = \beta = 0.3$. The output under Separate Accounting is in blue, and under Formula Apportionment in orange. It shows that the output in the high taxation jurisdiction is always larger under Separate Accounting than under Formula Apportionment. The output of the low taxation jurisdiction is also higher under Separate Accounting than under Formula Apportionment when the difference in corporate tax rates is very high. When the difference in corporate tax rates is moderate, the output in the low taxation jurisdiction is higher under Formula Apportionment than under Separate Accounting. As consumer surplus is proportional to the square of the output, we conclude that consumers in the high taxation jurisdiction always favor Separate Accounting over Formula Apportionment. Consumers in the low taxation jurisdiction prefer Formula Apportionment to Separate Accounting when the difference in corporate tax rates is moderate and Separate Accounting over Formula Apportionment when the difference over corporate tax rates is high.

Figure 11 compares the prices in region A under the two regimes, It shows that prices in the high taxation region are lower under Separate Accounting, but prices in the low taxation region are higher under Formula Apportionment.

Figure 12 pictures the relative output in region A, $\mu = \frac{x_A}{x_A + X_B}$, under the two regimes. It illustrates a striking difference between Separate Accounting and Formula Apportionment. Under Separate Accounting, the output of the platform is higher in the high taxation region, whereas under Formula Apportionment, the converse is true.

Finally, Figures 13 and 14 graph total output and the sum of consumer surpluses in the two regimes. They deliver a very strong message: Total output (and the sum of consumer surplus) is higher under Separate Accounting than under Formula Apportionment for all levels of taxation. The sum of consumer surplus is maximized under Separate Accounting when the difference in

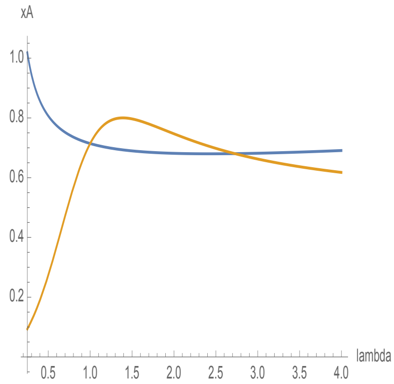

Figure 10: Output in Region A under SA and FA

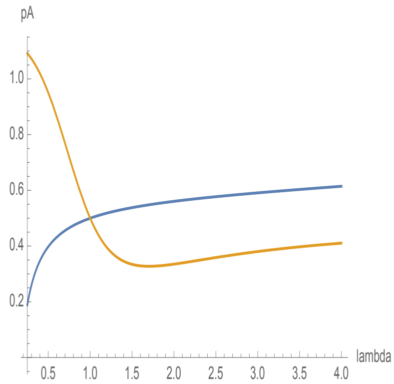

Figure 11: Price in Region A under SA and FA

corporate tax rates is highest, whereas it is highest under Formula Apportionment when the corporate tax rates are equal.

4 Profits and fiscal revenues

In this Section, we analyze how profits and fiscal revenues of the two countries are affected by the corporate tax rates.

4.1 Profits

The post-tax equilibrium profit is given by

$$\pi^* = r_B[\lambda x_A^*(1 - x_A^* + \beta x_B^*) + x_B^*(1 - x_B^* + \alpha x_A^*)].$$

Figure 15 graphs the platform's profit when $\alpha = \beta = 0.3$ under Separate Accounting (in blue) and Formula Apportionment (in orange). For a fixed level of taxation in region B, t_B,

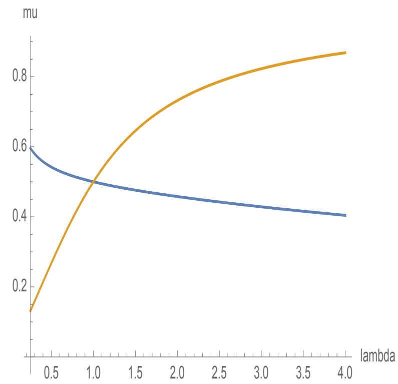

Figure 12: Relative outputs under SA and FA

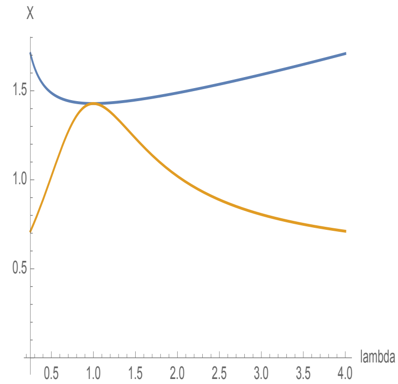

Figure 13: Total output under SA and FA

the post-tax profit of the platform is *decreasing* in the corporate tax rate t_A, hence increasing in λ. Figure 15 shows that the profit of the firm is always lower under Formula Apportionment than under Separate Accounting (except for the special case where $t_A = t_B$ when no distortions appear and the output choices are the same under the two régimes. Not surprisingly, the output distortions generated by Formula Apportionment result in a higher loss for the platform than the output distortions generated by Separate Accounting.

4.2 Tax revenues

Let t_B be the tax rate in jurisdiction B. The tax revenues in the two countries under SA are given by

$$\begin{aligned} T_A &= [(1-\lambda) + \lambda t_B] x_A^*(1 - x_A^* + \beta x_B^*), \\ T_B &= t_B x_B^*(1 - x_B^* + \alpha x_A^*) \end{aligned}$$

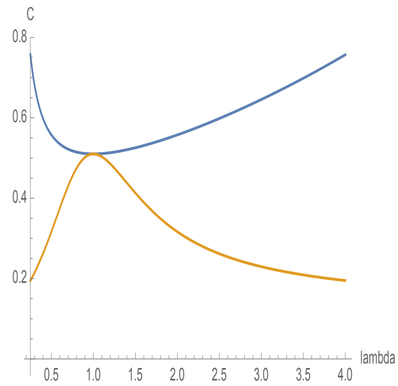

Figure 14: Consumer surplus under SA and FA

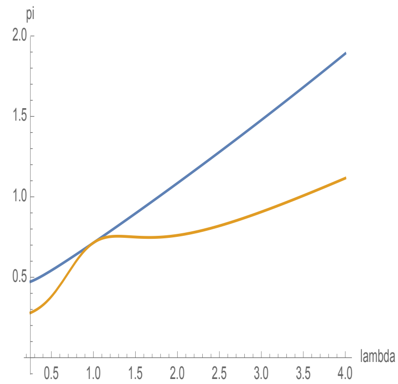

Figure 15: The platform's profit under SA and FA

and under FA by

$$T_A = [(1-\lambda)+\lambda t_B]\frac{x_A^*}{x_A^*+x_B^*}[x_A^*(1-x_A^*+\beta x_B^*)+x_B^*(1-x_B^*+\alpha x_A^*)],$$
$$T_B = t_B\frac{x_B^*}{x_A^*+x_B^*}[x_A^*(1-x_A^*+\beta x_B^*)+x_B^*(1-x_B^*+\alpha x_A^*)],$$

Figure **??** graphs the tax revenues of jurisdiction A when $\alpha = \beta = 0.3$ and $t_B = 0.3$ The blue line corresponds to the tax revenues under Separate Accounting and the orange line to the tax revenues under Formula Apportionment. We observe that the tax revenues of the high taxation country is higher under Separate Accounting than under Formula Apportionment but that the tax revenues of the low taxation country are higher under Formula Apportionment than under Separate Accounting. In fact, Formula Apportionment raises the share of taxes received by the low taxation country over the high taxation country, increasing the tax revenues of the low taxation country. This suggests that two country with different corporate tax rates would disagree on the best formula for profit sharing.

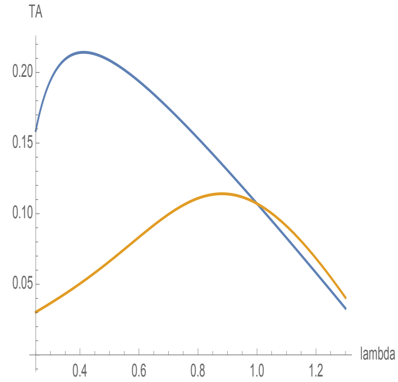

Figure 16: Tax revenues of jurisdiction A under SA and FA

4.3 Fiscal competition

We now exploit the intuition obtained from the Figures illustrating the tax revenues of the two countries to solve for the symmetric equilibrium of a model of fiscal competition between the two jurisdictions. Consider a model with symmetric externalities $\alpha = \beta$. Let the two jurisdictions A and B choose the corporate tax rates t_A and t_B simultaneously to maximize tax revenues T_A and T_B. A *symmetric* equilibrium of the model of tax competition is a corporate tax rate t^* such that

$$\frac{\partial T_A}{\partial \lambda}|t_B = t^*, \lambda = 1 = 0.$$

Now in the case of Separate Accounting

$$\begin{aligned}\frac{\partial T_A}{\partial \lambda}|t_B = t^*, \lambda = 1 &= -(1-t^*)[x_A^*(1 - x_A^* + \beta x_B^*)] \\ &+ t^* \frac{\partial (x_A^*(1 - x_A^* + \beta x_B^*))}{\partial \lambda}|_{\lambda=1}.\end{aligned}$$

Now,

$$\frac{\partial (x_A^*(1 - x_A^* + \beta x_B^*))}{\partial \lambda}| - \lambda = 1 = (1 - 2x_A^* + \beta x_B^*)\frac{\partial x_A^*}{\partial \lambda} + \beta x_A^* \frac{\partial x_B^*}{\partial \lambda}.$$

Furthermore, at $\lambda = 1$, $(1 - 2x_A^* + \beta x_B^*) = 0$ so that

$$\frac{\partial (x_A^*(1 - x_A^* + \beta x_B^*))}{\partial \lambda}| - \lambda = 1 = \beta x_A^* \frac{\partial x_B^*}{\partial \lambda}.$$

Recall that, under Separate Accounting, $\frac{\partial x_B^*}{\partial \lambda}|_{\lambda=1} > 0$ so that there exists an interior symmetric equilibrium tax rate $t_{SA}^* > 0$.

In the case of Formula Apportionment

$$\begin{aligned}\frac{\partial T_A}{\partial \lambda} &= -(1-t^*)[\frac{x_A^*}{x_A^*+x_B^*}V] \\ &+ t^*\frac{\partial \mu^*}{\partial \lambda}V \\ &+ t^*\mu\frac{\partial V}{\partial \lambda}_{\lambda=1}.\end{aligned}$$

Now

$$\frac{\partial V}{\partial_\lambda} = \frac{\partial V}{\partial x_A^*}\frac{\partial x_A^*}{\partial \lambda} + \frac{\partial V}{\partial x_B^*}\frac{\partial x_B^*}{\partial \lambda}.$$

In addition, when $\lambda = 1$, $\frac{\partial V}{\partial x_A^*} = \frac{\partial V}{\partial x_B^*} = 0$,
so that

$$\frac{\partial T_A}{\partial \lambda} = -(1-t^*)[\mu^* V] + t^*\frac{\partial \mu^*}{\partial \lambda}_{\lambda=1}V.$$

Now, using L'Hopital's rule, we compute $\frac{\partial \mu^*}{\partial \lambda}_{\lambda=1} = \frac{3}{8}$ so that $t_{FA}^* = \frac{1}{2}\frac{8}{7} = \frac{4}{7}$.

We summarize these findings in the following Proposition

Proposition 4 *There exists a symmetric equilibrium of the model of fiscal competition with interior corporate tax rate both under Separate Accounting and under Formula Apportionment,*

$$\begin{aligned}t_{SA}^* &= \frac{2+2\alpha}{2+4\alpha-2\alpha^2}, \\ t_{FA}^* &= \frac{4}{7}\end{aligned}$$

Note that the equilibrium corporate tax rate is decreasing in α under Separate Accounting, reaching the value $\frac{8}{11}$ when $\alpha = \frac{1}{2}$. Under Formula Apportionment, the equilibrium tax rate is independent of the level fo externalities, identically equal to $\frac{4}{7} < \frac{8}{11}$. Hence the equilibrium corporate tax rate is always lower under Formula Apportionment than under Separate Accounting, reflecting a higher level of competition between the jurisdictions.

5 Extensions

5.1 Beyond the linear model

When demand is not linear, computing the effect of changes in tax rates on output under Separate Accounting and Formula Apportionment becomes intractable, except locally around the point where the two tax rates are equal. We thus consider the local effects of a change in tax rate around the point $\lambda = 1$, assuming furthermore that the inverse demand functions on the two markets are equal.

5.1.1 Separate accounting

Under *Separate Accounting (SA)*, the platform declares separately the profit made in each jurisdiction, and pays taxes in each jurisdiction based on this profit. This amounts to apportioning profit based on the profit made in each jurisdiction. The post-tax profit of the platform is then given by

$$\Pi = (1 - t_A)V_A + (1 - t_B)V_B.$$

The optimal choice of the platform is characterized by the first order conditions

$$(1 - t_A)\frac{\partial V_A}{\partial x_A} + (1 - t_B)\frac{\partial V_B}{\partial x_A} = 0, \quad (3)$$

$$(1 - t_A)\frac{\partial V_A}{\partial x_B} + (1 - t_B)\frac{\partial V_B}{\partial x_B} = 0. \quad (4)$$

We use the first order conditions on the profit Π (**??**) and (**??**) to compute

$$\frac{\partial \phi_A}{\partial t_A} = \frac{\frac{\partial V_A}{\partial x_A}}{\frac{\partial^2 \Pi}{\partial x_A \partial x_A}},$$

$$\frac{\partial \phi_B}{\partial t_A} = \frac{\frac{\partial V_A}{\partial x_B}}{\frac{\partial^2 \Pi}{\partial x_B \partial x_B}},$$

$$\frac{\partial \phi_A}{\partial x_B} = -\frac{(1 - t_A)\frac{\partial^2 V_A}{\partial x_A \partial x_B} + (1 - t_B)\frac{\partial^2 V_B}{\partial x_A \partial x_B}}{\frac{\partial^2 \Pi}{\partial x_A \partial x_A}},$$

$$\frac{\partial \phi_B}{\partial x_A} = -\frac{(1 - t_A)\frac{\partial^2 V_A}{\partial x_A \partial x_B} + (1 - t_B)\frac{\partial^2 V_B}{\partial x_A \partial x_B}}{\frac{\partial^2 \Pi}{\partial x_B \partial x_B}}$$

Assuming that the post-tax profit is concave, $\frac{\partial^2 \Pi}{\partial x_A \partial x_A}$ and $\frac{\partial^2 \Pi}{\partial x_B \partial x_B}$ are both negative. In addition, because externalities across markets are positive, the cross derivatives $\frac{\partial^2 V_A}{\partial x_A \partial x_B}$ and $\frac{\partial^2 V_B}{\partial x_A \partial x_B}$ are both positive. Hence $\frac{\partial \phi_A}{\partial x_B}$ and $\frac{\partial \phi_B}{\partial x_A}$ are both positive: the optimal choice of x_A is increasing in x_B and the optimal choice of x_B is increasing in x_A.

Because externalities are positive, $\frac{\partial V_A}{\partial x_B} \geq 0$, so that $\frac{\partial \phi_B}{\partial t_A} \leq 0$: the direct effect of an increase in t_A on the optimal choice of x_B is negative. This effect is easy to interpret: as the profits in country A are more heavily taxed, the positive external effect of an increase in x_B on the profit of the platform in jurisdiction A is reduced, resulting in a decrease in the optimal value of x_B. Finally, at the optimum, $\frac{\partial V_A}{\partial x_A} \leq 0$, so that $\frac{\partial \phi_A}{\partial t_A} \geq 0$: an increase in t_A results in an increase in the number of participants in country A. This comparative statics effect is somewhat surprising, as it implies that the platform expands its coverage in the country with the higher tax rate. It is however easy to interpret. As the tax rate t_A increases, the platform has an incentive to shift profit from jurisdiction A to jurisdiction B. In order to do so, it increases its coverage in market A, resulting in higher demand in market B through the positive cross-market externality. When the difference between tax rates becomes sufficiently high, and if the external effects are sufficiently strong, the platform may even have an incentive to shift all its profit from jurisdiction A to jurisdiction B, by setting a zero price for its service in the high tax country, expanding coverage there in order to increase demand and profit in the low tax country.

On balance, we thus find that the direct and indirect effects of an increase in t_A on x_A and x_B operate in opposite directions. An increase in t_A leads to a direct expansion of x_A and a direct reduction in x_B, but indirectly results in a decrease in x_A and an increase in x_B. However, in the natural situation where external effects are not too large, it is reasonable to assume that the direct effect dominates the indirect effect so that output x_A increases in t_A and output x_B decreases in t_A.

5.1.2 Formula Apportionment

In the régime of *Formula Apportionment (FA)*, the profit is apportioned according to the number of users in the two jurisdictions. The post-tax profit of the platform is given by

$$\Pi = V(1 - t_A \frac{x_A}{x_A + x_B} - t_B \frac{x_B}{x_A + x_B}).$$

The optimal choice of the platform is characterized by the first order conditions

$$\frac{\partial V}{\partial x_A}(1 - t_A \frac{x_A}{x_A + x_B} - t_B \frac{x_B}{x_A + x_B}) - V \frac{(t_A - t_B)x_B}{(x_A + x_B)^2} = 0, \quad (5)$$

$$\frac{\partial V}{\partial x_B}(1 - t_A \frac{x_A}{x_A + x_B} - t_B \frac{x_B}{x_A + x_B}) + V \frac{(t_A - t_B)x_A}{(x_A + x_B)^2} = 0. \quad (6)$$

Hence, if $t_A > t_B$,

$$\frac{\partial V}{\partial x_A} > 0,$$

$$\frac{\partial V}{\partial x_B} < 0.$$

As the pre-tax profit is concave, we conclude that when the tax rate is higher in country A than in country B, the number of users chosen by the platform is lower in country A than in country B. We use the first order conditions on the profit Π under FA (**??**) and (**??**) to compute

$$\frac{\partial \phi_A}{\partial t_A} = \frac{\frac{x_A}{x_A+x_B}\frac{\partial V}{\partial x_A} + \frac{V x_B}{(x_A+x_B)^2}}{\frac{\partial^2 \Pi}{\partial x_A \partial x_A}},$$

$$\frac{\partial \phi_B}{\partial t_A} = \frac{\frac{x_A}{x_A+x_B}\frac{\partial V}{\partial x_B} - \frac{V x_A}{(x_A+x_B)^2}}{\frac{\partial^2 \Pi}{\partial x_B \partial x_B}},$$

$$\frac{\partial \phi_A}{\partial x_B} = -\frac{\frac{\partial^2 V}{\partial x_A \partial x_B}(1 - T(x_A, x_B)) - [\frac{\partial V}{\partial x_A}\frac{\partial T}{\partial x_B} + \frac{\partial V}{\partial x_B}\frac{\partial T}{\partial x_A}] - V\frac{\partial^2 T}{\partial x_A \partial x_B}}{\frac{\partial^2 \Pi}{\partial x_A \partial x_A}},$$

$$\frac{\partial \phi_B}{\partial x_A} = -\frac{\frac{\partial^2 V}{\partial x_A \partial x_B}(1 - T(x_A, x_B)) - [\frac{\partial V}{\partial x_A}\frac{\partial T}{\partial x_B} + \frac{\partial V}{\partial x_B}\frac{\partial T}{\partial x_A}] - V\frac{\partial^2 T}{\partial x_A \partial x_B}}{\frac{\partial^2 \Pi}{\partial x_B \partial x_B}}$$

The expressions for Formula Apportionment are more complex than for separate accounting By concavity, $\frac{\partial^2 \Pi}{\partial x_A \partial x_A}$ and $\frac{\partial^2 \Pi}{\partial x_B \partial x_B}$ are both negative. Let $t_A \geq t_B$. $\frac{\partial V}{\partial x_A} \geq 0$, $\frac{\partial \phi_A}{\partial t_A} \leq 0$. The direct effects can be signed easily: $\frac{\partial \phi_A}{\partial t_A} \leq 0$ and $\frac{\partial \phi_B}{\partial t_A} \geq 0$.

The signs of $\frac{\partial \phi_A}{\partial x_B}$ and $\frac{\partial \phi_B}{\partial x_A}$ are those of the numerators of the fractions, which are identical.[2] This identical sign cannot be ascertained in general. The first term is positive. The second term is also positive since using the first order condition: $\frac{\partial V}{\partial x_A}\frac{\partial T}{\partial x_B} = \frac{1}{1-T(x_A,x_B)}\frac{\partial T}{\partial x_A}\frac{\partial T}{\partial x_B}$, which is negative. For the third term, compute $\frac{\partial^2 T}{\partial x_A \partial x_B} = \frac{(t_A-t_B)(x_A-x_B)}{(x_A+x_B)^3}$.

When t_A is close to t_B, $\frac{\partial \phi_A}{\partial x_B}$ converges to $-\frac{(1-\frac{t_A x_A}{x_A+x_B}-\frac{t_B x_B}{x_A+x_B})\frac{\partial^2 V}{\partial x_A \partial x_B}}{\frac{\partial \phi_A}{\partial x_A}}$, which is positive. Similarly, $\frac{\partial \phi_B}{\partial x_A}$ converges to the positive term $-\frac{(1-\frac{t_A x_A}{x_A+x_B}-\frac{t_B x_B}{x_A+x_B})\frac{\partial^2 V}{\partial x_A \partial x_B}}{\frac{\partial \phi_A}{\partial x_A}}$ when t_A approaches t_B. Its sign depends on the demand. However, if demands are symmetric it is likely to be negative, yielding a positive numerator for sure. Furthermore, when t_A is close to t_B, the first term of the numerator dominates, because the second and the third are proportional to $(t_A - t_B)$. Hence, both $\frac{\partial \phi_A}{\partial x_B}$ and $\frac{\partial \phi_B}{\partial x_A}$ are likely to be positive. As in the case of SA, the direct and indirect effects are likely to have opposite signs. Assuming that the direct effect dominates the indirect effect, we obtain that an increase in t_A leads to a reduction in the number of participants in country A and an increase in the number of participants in country B. To understand this effect, notice that the platform has an incentive to reduce the number of users in the high tax country and increase the number of users in the low tax country in order to shift profit to the low tax jurisdiction.

6 Conclusion

This paper analyzes taxation of a two-sided platform attracting users from different jurisdictions under two profit-sharing régimes: separate accounting and formula apportionment based on the

[2]This holds true generally as both have the sign of $\frac{\partial^2 \Pi}{\partial x_B \partial x_B}$.

number of users in the two countries. When corporate income tax rates are different in the two jurisdictions, the platform distorts prices and outputs in order to shift profit to the low tax country. When cross effects are present on both sides of the market, the platform has an incentive to increase output in the high tax country and decrease output in the low tax country under separate accounting. Under formula apportionment, the incentives are reversed, and the platform reduces output in the high tax country and increases output in the low tax country. We show that separate accounting always dominates formula apportionment for the platform, but that consumer surplus and tax revenues may be higher under formula apportionment than under separate accounting. In particular, consumers in the high tax country always favor separate accounting, whereas consumers of the low tax country prefer formula apportionment when the difference in corporate tax rates is not too high. Fiscal revenues of the high tax country are higher under Separate Accounting and fiscal revenues of the low tax country are higher under Formula Apportionment. Finally, we compute the equilibrium corporate tax rates under Separate Accounting and Formula Apportionment in a symmetric model of fiscal competition.

While this analysis is a first step in the study of multinational two-sided platforms, we are aware of important limitations that need to be addressed in future research. First, because users are immobile, profit shifting does not arise under separate accounting, and tax competition between the two jurisdictions is weak. In order to introduce profit shifting and tax competition, we need to allow users to move across jurisdictions in response to the pricing decisions of the platform. Second, our model is too simplistic to account for existing global value chains of internet platforms. We do not consider for example incentives to improve the matching algorithm and the geographical distribution of intellectual property rights. In order to get a better understanding of the location decisions of internet platforms, we need to include these elements in future research.

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