OPTIMIZATION

Course title – Intitulé du cours	Optimization
Level / Semester – Niveau /semestre	M2/S1
School – Composante	Ecole d'Economie de Toulouse
Teacher – Enseignant responsable	J. Bolte & T. Mariotti
Other teacher(s) – Autre(s) enseignant(s)	Amirreza Ahmadzadeh
Other teacher(s) – Autre(s) enseignant(s)	
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Other teacher(s) – Autre(s) enseignant(s)	
Lecture Hours – Volume Horaire CM	30 :00
TA Hours – Volume horaire TD	15 :00
TP Hours – Volume horaire TP	
Course Language – Langue du cours	English
TA and/or TP Language – Langue des TD et/ou TP	English

Teaching staff contacts – Coordonnées de l'équipe pédagogique :

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Course Objectives – Objectifs du cours :

The goal of these series of lectures is to provide M2 students and future PhD with a solid background in Optimization. In particular the course will emphasize rigorous methods, namely: importance of proofs, assumption checking, heuristics (``ideas genesis"). Such tasks become easier when the notions at stake are associated to concrete situations, that is why we shall, whenever possible, attach to each introduced notion geometrical ideas or concrete examples. Applications to Economics will be provided.

An indicative outline is as follows (in gray the material is provided but not studied)

1. Static Optimization

- Theoretical tools: projection theorem, separation results, Riesz representation theorem, convex cones, Farkas lemma
- Convex functions: continuity issues, characterizations of convexity, existence and characterization of minimizers, second-order conditions for minimization
- Convex programming : Lagrange method and KKT conditions, Lagrange multipliers,
- Lagrangian duality
- Sensitivity and duality

- Nonlinear programming : KKT conditions

2. Introduction to the calculus of variations

- Necessary conditions : Euler-Lagrange conditions, transversality condition
- Regularity of the solutions: continuously differentiable solutions, piecewise continuously differentiable solutions, Erdmann-Weierstrass corner condition
- Second order conditions and Legendre theorem
- Infinite-horizon problem
- Examples

3. Introduction to optimal control

• Pontryagine maximum principle
• Sufficient conditions
• Dynamic programming: Bellman principle

• Infinite-horizon problems

Prerequisites – Pré requis : The outline of the course given above is quite ambitious in terms of length. It corresponds to an ideal situation in which most of the students are already familiar with elementary concepts of Calculus/Analysis such as: closed/compacts sets, continuity, convexity, etc. In other words, some mathematical maturity from the students will help them to better appreciate the course and will allow us to go further.

A background in elementary linear algebra and calculus is strongly advised

Practical information about the sessions – Modalités pratiques de gestion du cours :

Questions and discussions on the lectures are highly encouraged

Grading system – Modalités d'évaluation :

There will be a two-hours final exam which is meant to check fundamental knowledge and basic proficiencies. More difficult questions requiring some originality or ingenuity will be also included, they represent between 20% and 30% of the total mark.

Bibliography/references – Bibliographie/références :

• Most accessible references
∗ Advanced books for rather experienced students or for advanced questions

Convex Analysis and Optimization

– * Bauschke, Heinz H.; Combettes, Patrick L. Convex analysis and monotone operator theory in Hilbert spaces. CMS Books in Mathematics/Ouvrages de Math´ematiques de la SMC. Springer, New

York, 2011. xvi+468 pp.
– Berge, C., “Espaces topologiques et fonctions multivoques”, Dunod, Paris, 1959. (One of the most original and motivating book in this list)

– Barvinok, A first course in Convex Analysis1 AMS (A must for duality theory in infinite dimensional spaces)
– Bonnans F., “Optimisation continue”, Dunod, 2006.
– Borwein, J., Lewis, A.S., “Convex Analysis and Nonlinear Optimization”, Springer-Verlag 2000.
– • Ciarlet, P.-G, “Introduction à l’analyse matricielle et à l’optimisation”, Masson, Paris, 1988.
– * Hiriart-Urruty, J.-B, Lemaŕechal, C., “Convex analysis and minimization algorithms”, Springer-Verlag, 1996. [algorithmic oriented]
– Rockafellar, R.T., “Convex analysis”, Princeton University Press, 1970.
– • Sundaram, R. K. “A first course in optimization theory”. Cambridge University Press, Cambridge, 1996. xviii+357 pp.

Fixed point theory

– Border, K.C., “Fixed point theorems with applications to economics and game theory”, Cambridge University Press, 1985. (A ”must” for fixed-point like results)

Calculus of variations and Optimal control

– Gelfand I.M. and Fomin S.V., “Calculus of variations and Optimal control”, Dover publications, 1963.

– * Clarke, Frank H., “Functional Analysis, Calculus of Variations and Optimal Control”, Springer, 2013.

– * Clarke, Frank H. Optimization and nonsmooth analysis. Canadian Mathematical Society Series of Monographs and Advanced Texts. A Wiley-Interscience Publication. John Wiley & Sons, Inc., New York, 1983. xiii+308 pp.

– Dacorogna, B., “Introduction to the calculus of variations”. Translated from the 1992 French original. Second edition. Imperial College Press, London, 2009. xiv+285 pp.

– • Demange, G., Rochet, J.-C, “Méthodes mathématiques de la finance”, Economica, 1992.

– Evans, L.C, (Berkeley), Online course on Optimal Control at:

http://math.berkeley.edu/evans/control.course.pdf

– • Luenberger, D. G. “Optimization by vector space methods”. John Wiley & Sons, Inc., New York-London-Sydney 1969 xvii+326 pp

– • Liberzon, D., Calculus of Variations and Optimal Control Theory: A Concise Introduction, Princeton university press, 2012.

– • Weber, Thomas A., Optimal Control Theory with Applications in Economics, MIT Press, 2011.

Session planning – Planification des séances

To be announced further

Distance learning – *Enseignement à distance :*

Distance learning can be provided when necessary by implementing:

- Interactive virtual classrooms
- Recorded lectures (videos)
- MCQ tests and other online exercises / assignments
- Remote (online) tutorials (classes)
- Chatrooms

En cas de nécessité, un enseignement à distance sera assuré en mobilisant:

- Classe en ligne interactive
- Vidéo enregistrée de la présentation du matériel pédagogique
- QCM et exercices en ligne
- TP/TD à distance
- Forum...